

INSTITUTO POLITÉCNICO NACIONAL ESCUELA SUPERIOR DE FÍSICA Y MATEMÁTICAS

Amigo 친구 global games



Questions

1 Let $F=\{1,2,3,4,5\}$. Build a computer program that computes a bijection between the set of natural numbers and the set of finite sequences from F (i.e., a program that, with input a natural number, outputs a finite sequence of numbers between 1 and 5; in addition to the program itself, you have to include in the supporting documentation some justification for why the function given by your program is a bijection).

2 Let $Z(k)=\sum_{n=1}^{infty} \frac{1}{n^k}$ be the Riemann zeta function. Show that $\sum_{n=k}^{infty} (-1)^{n+1}{n \cdot k} (Z(n+1)-1)$ converges.

3 Let us assume that we have functions f,g,h and constants x,y such that for any expressions a, b, we have

f(a,x) = x, g(y,a) = y, g(a,x) = a, f(a,y) = a, h(x) = y,h(y) = x,

write a program that given three positive integers, n, m, q, it evaluates $f^n(g^m(x, h^q(x)), h^n(+m)(x))$.

Note: Expressions are assumed to be constructed from applications of functions to constants.